

14/04/20

## Exercise - 1

There are various different types of number system. The different types are as follow -

(i) Natural Numbers - these numbers are used for counting and always start with one to infinitely.

Example - 1, 2, 3, 4, ...

(ii) Whole Numbers - this is the set of all natural numbers including 0 to infinitely.

Example - 0, 1, 2, 3, 4, ... ∞

(iii) Integers - the set of all whole numbers and their respective negative number is called integers.

(iv) Rational number - such numbers which and their respective negative numbers is called the integers can be express in the form of  $\frac{p}{q}$  where  $p$  and  $q$  both are integers and  $q \neq 0$ , are called Rational number.

(v) Irrational number - An irrational number is real no. that cannot be expressed as a ratio of two integers.

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Ex. "π" or "Pi" (3, 14, 15, 9)

(vi) Terminating decimal → A terminating decimal is usually defined as a decimal number that contains a finite number of digits after the decimal point.

Example →  $\frac{1}{4} = \sqrt[4]{10} = 0.25$

$$\begin{array}{r} 10 \\ 4 \overline{) 10} \\ \underline{-8} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

(vii) Non-terminating decimal - A non-terminating, non-repeating decimal is a decimal number that continues endlessly, with no group of digits repeating endlessly. Decimals of this type cannot be represented as fractions, and as a result are irrational numbers.

Example -  $\frac{6}{7} = 0.857142$

### Definitions -

Natural no. → These no. starts from 1 to infinity

Whole no. → These no. starts from 0 to infinity

Integers → It include natural no. and their inverse

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Rational no.  $\rightarrow$  The no. can be expressed in the form of  $p/q$ .

Irrational no.  $\rightarrow$  These no. cannot be expressed in the form of  $p/q$ .

Real no.  $\rightarrow$  It contains all rational and irrational no. that can be represented on a no. line.

Prime no.  $\rightarrow$  These no. are greater than 1 only factors are 1 and itself.

Composite no.  $\rightarrow$  These no. having factors more than factor two.

Ex. 1.1

Q: 1. Is zero a rational no? can you write in the form  $\frac{p}{q}$ , where p and q are integers and  $q \neq 0$ ?

Ans: Yes, zero is rational number

Ex.  $\rightarrow \frac{0}{1}, \frac{0}{2}, \frac{0}{3}, \frac{0}{40}$

Q: 2. Find six rational number between 3 and 4

Ans: There were infinite rational number between 3 and 4.

$$\frac{3}{1} \times \frac{7}{7} = \frac{21}{7}$$

$$\frac{4}{1} \times \frac{7}{7} = \frac{28}{7}$$

$$\left[ \frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7} \right]$$

Q: 3. Find the five rational no. between  $\frac{3}{5}$  and  $\frac{4}{6}$ .

Ans: There were infinite rational no. are present between  $\frac{3}{5}$  and  $\frac{4}{6}$ .

$$\frac{3}{5} \times \frac{6}{6} = \frac{18}{30}$$

$$\frac{4}{6} \times \frac{6}{6} = \frac{24}{30}$$

$$\left[ \frac{19}{30}, \frac{20}{30}, \frac{21}{30}, \frac{22}{30}, \frac{23}{30} \right]$$

Q: 4. State wheather the following statements are true or false.

(i) Every natural no. is a whole no.  
Ans → True.

Natural no. → 1, 2, 3, ...

Whole no. → 0, 1, 2, 3, ...

(ii) Every integer is a whole no.  
Ans → False.

Integers → ... -2, -1, 0, 1, 2, ... They some

integers are negative.

Whole no. → 0, 1, 2, 3, ... They all are positive.

(iii) Every rational no. is a whole no.

Ans → False.

Ex. →  $\frac{p}{q}$  = 3.33 which is not a whole no.

### Exercise-1.2

Q.1 State whether the following statements are true or false. Justify your answer.

(i) Each irrational no. is a real no.

Ans → Every no. which we can show on line is real no. and irrational no. can be expressed on no. line. So, it is True.

(ii) Every point on the number line is of the form  $\sqrt{m}$ , where  $m$  is a natural number.

Ans → False because natural no. are never negative.

(iii) Every real number is an irrational no.  
False.

Q. 2: Are the square roots of all positive integers irrational? If not, give an ex. of the square root of a no. that is a rational no.  
Ans: No, The square roots of all positive integers is not irrational.

Ex.  $\rightarrow \sqrt{4}, \sqrt{9}, \sqrt{36}, \sqrt{49}$   
 $\quad \quad \quad \quad \quad \sqrt{2 \times 2} \quad \quad \sqrt{3 \times 3} \quad \quad \sqrt{6 \times 6} \quad \quad \sqrt{7 \times 7}$   
 $\quad \quad \quad \quad \quad = 2 = \frac{2}{1} \quad \quad \Rightarrow 3 \quad \quad = 6 \quad \quad = 7$

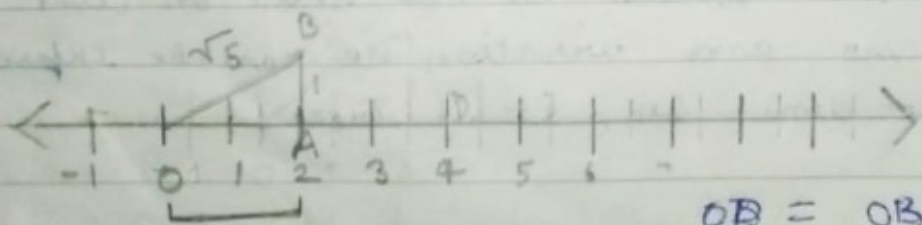
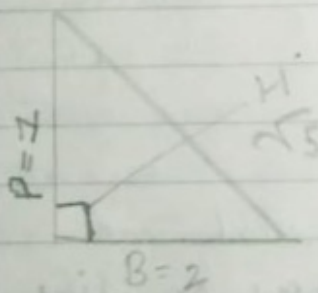
Q. 3: Show how  $\sqrt{5}$  can be represented on the number line.

By Pythagoras Th.

$$H = \sqrt{P^2 + B^2}$$

$$H = \sqrt{1^2 + 2^2}$$

$$H = \sqrt{1 + 4} = \sqrt{5}$$

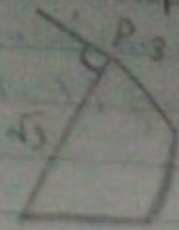


$$OB = OB = \sqrt{5}$$

$$B = 1$$

$$P = 2$$

Q.1 classroom activity (construct the square root spiral)



$$\begin{aligned} OP &= \sqrt{1} \\ OP^2 &= \sqrt{2} \\ OP^4 &= \sqrt{3} \\ OP^5 &= \sqrt{4} \\ OP^6 &= \sqrt{5} \\ OP^7 &= \sqrt{6} \end{aligned}$$

### Ex 1.3

Terminating decimal expansion - In this our remainder becomes zero after some times.

$$\frac{42}{100}$$

Non-Terminating Recurring - In this remainder repeats after some time.

$$\frac{1}{3}$$

Non-Terminating non-recurring - In this remainder never zero not repeat.

Q.2 Write the following in decimal form and say what kind of decimal expansion each has:

(i)  $\frac{36}{100}$        $100 \overline{) 360} \quad 0.36$  - Terminating decimal.

$$\begin{array}{r} 36 \\ 100 \overline{) 360} \\ \underline{300} \\ 600 \\ \underline{600} \\ 0 \end{array}$$

vi

$$\frac{329}{400} =$$

$$400 \overline{) 3290.08}$$

$$\underline{- 3200}$$

$$60900$$

$$\underline{- 800}$$

$$1000$$

$$\underline{- 800}$$

$$2000$$

$$\underline{- 2000}$$

$$0$$

Terminating decimal

Q. 2 you know that  $\frac{1}{7} = 0.\overline{142857}$ . Can you predict

what the decimal expansion of  $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}$  are, without doing the long

division, if so, how?

$$\frac{1}{7} = \overline{0.142857}$$

$$\frac{2}{7} = 2 \times \frac{1}{7} = 2 \times 0.142857 = \overline{0.285714}$$

$$\frac{3}{7} = 3 \times \frac{1}{7} = 3 \times 0.142857 = \overline{0.428571}$$

$$\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 0.142857 = \overline{0.571428}$$

$$\frac{5}{7} = 5 \times \frac{1}{7} = 5 \times 0.142857 = \overline{0.714285}$$



$$\frac{6}{7} = \frac{6 \times 1}{7} = 6 \times 0.142857 = \frac{6 \times 157142}{1000000}$$

Q:3 Express the following in the form of  $\frac{p}{q}$ , where  $p$  and  $q$  are integers and  $q \neq 0$ .

(i)  $0.\overline{6}$

Ans  $\rightarrow$  Let  $x = 0.\overline{666}$

Multiply by 1000 both side (i)

$$10x = 10 \times 0.\overline{666}$$

$$10x = 6.\overline{666} \quad (ii)$$

sub eq (i) from (ii)

$$10x = 6.\overline{666}$$

$$x = 0.\overline{666}$$

$$9x = 6.0$$

$$9x = 6$$

$$x = \frac{6}{9} = \frac{2}{3}$$

$$0.\overline{6} = \frac{2}{3} \text{ Ans.}$$

(ii)  $0.\overline{47}$

$$0.\overline{47} = 0.4777$$

Let  $x = 0.\overline{4777} \quad (i)$

Multiply by, both side.

$$10x = 4.\overline{777} \quad (ii)$$

sub both eq. (i) from (ii)

$$10x = 4.\overline{777}$$

$$10x = 4.777$$

$$x = 0.4777$$

$$9x = 4.30$$

$$9x = 43$$

$$9x = \frac{43}{10}$$

$$x = \frac{43}{10 \times 9}$$

$$x = \frac{43}{90}$$

$$0.\overline{47} = \frac{43}{90}$$

(iii)  $0.\overline{001}$

Ans  $\rightarrow 0.001 = 0.001001 \dots$

Let  $x = 0.\overline{001} = 0.001001 \dots$  (1)

Multiply by 1000 both.

$$1000x = 1.001001 \dots$$

Sub (1) from (2)

$$\begin{array}{r} 1000x = 1.001001 \dots \\ x = 0.001001 \dots \\ \hline 999x = 1.0 \end{array}$$

$$999x = 1$$

$$x = \frac{1}{999}$$

$$0.001 = \frac{1}{999} \text{ Ans.}$$

Q: Express  $0.99909$  in the form of  $\frac{p}{q}$ .  
Are you surprised by your answer? With your teacher and classmates discuss why the answer make sense.

Ans:  $0.\bar{9} = 0.999\dots$

Let  $x = 0.999\dots \rightarrow \textcircled{1}$

Multiply by 10 both sides

$10x = 9.999\dots \rightarrow \textcircled{10}$

Subtract eq.  $\textcircled{1}$  from  $\textcircled{10}$

$$10x = 9.999\dots$$

$$x = 0.999\dots$$

$$9x = 9.0$$

$$9x = 9$$

$$x = \frac{9}{9} = 1 \text{ Answer.}$$

Q: What can the maximum no. of digits be in the repeating block of digits in decimal expansion of  $\frac{1}{17}$ ? Perform the division to check your answer.

Ans:  $17 \overline{) 100} 0.0588235294117647$

$$\begin{array}{r}
 100 \\
 -85 \\
 \hline
 150 \\
 -136 \\
 \hline
 140 \\
 -136 \\
 \hline
 40 \\
 -34 \\
 \hline
 60
 \end{array}$$

$$\begin{array}{r}
 -51 \\
 \hline
 90 \\
 -85 \\
 \hline
 50 \\
 -34 \\
 \hline
 160 \\
 -153 \\
 \hline
 70 \\
 -68 \\
 \hline
 20 \\
 -17 \\
 \hline
 30 \\
 -17 \\
 \hline
 130 \\
 -119 \\
 \hline
 110 \\
 -102 \\
 \hline
 80 \\
 -68 \\
 \hline
 120 \\
 -119 \\
 \hline
 1
 \end{array}$$

Ans. =  $0.\overline{0581235294117147} = 16 \text{ blocks}$

Q:6 Look at the several ex. of rational no. in the form of  $p/q$  ( $q \neq 0$ ), where  $p$  and  $q$  are integers with no common factors other than 1 and having terminating decimal representation (expansion). Can you guess what property  $q$  must satisfy?

ans  $\rightarrow$  Example =  $\frac{2}{5} = 0.4$

$$\frac{5}{2} = 2.5$$

$$\frac{3}{5} = 0.6$$

Q must satisfy the factors of 2 and 5.

$$\frac{5}{3} = 1.6666 \dots$$

Q:7 Write 3 no. whose decimal expansion are non Terminating Non-Recurring.

ans →  $\pi = 3.14159 \dots$

$$\sqrt{2} = 1.414 \dots$$

$$\sqrt{3} = 1.7 \dots$$

$$0.101001000 \dots$$

Q:8 Find 3 different irrational no. between the rational no.  $\frac{5}{7}$  and  $\frac{9}{11}$ .

ans →  $\frac{5}{7} = 0.714285 \dots$

$$\frac{9}{11} = 0.81 \dots$$

$$= 0.715889 \dots$$

$$= 0.73429 \dots$$

$$= 0.7692, 0.77912 \dots$$

} Irrational no.

Q:9 Classify the following no. as rational or irrational

(i)  $\sqrt{23}$

ans →  $\sqrt{23} = 4.795831 \dots$

It is an irrational no.

(ii)  $\sqrt{225} = 15.15$

ans → Therefore  $\sqrt{225}$  is a rational no.

(iii - iv - v) are at p.no. -

Ex. - 1.3. (Remaining q. - iii, iv, v)

(iii) 0.3796.

Ans → It is a terminating decimal. Therefore, it is a rational no.

(iv) 7.478478...

Ans → The given no 7.478478... is non-terminating recurring decimal, which can be converted into  $\frac{p}{q}$  form. While converting 7.478478... into  $\frac{p}{q}$  form, we get

$$x = 7.478478... \quad (a)$$

$$1000x = 7478.478478... \quad (b)$$

While subtracting a from b, we get

$$1000x = 7478.478478...$$

$$- x = 7.478478...$$

$$999x = 7471 \quad 999x = 7471 \quad x = \frac{7471}{999}$$

Therefore, 7.478478 is a rational no.

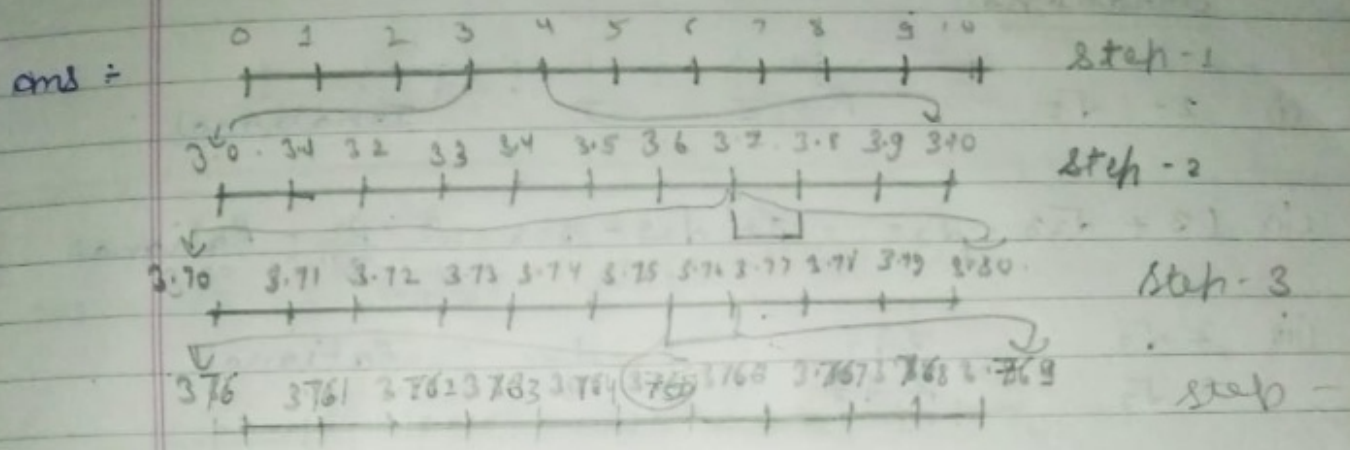
(v) 1.101001000100001...

Ans → We can observe that the no 1.101001000100001... is a non-terminating non-recurring decimal. Thus, non-terminating or non-recurring decimal cannot be converted into  $\frac{p}{q}$  form. Therefore, we conclude that 1.101001000100001... is an irrational no.

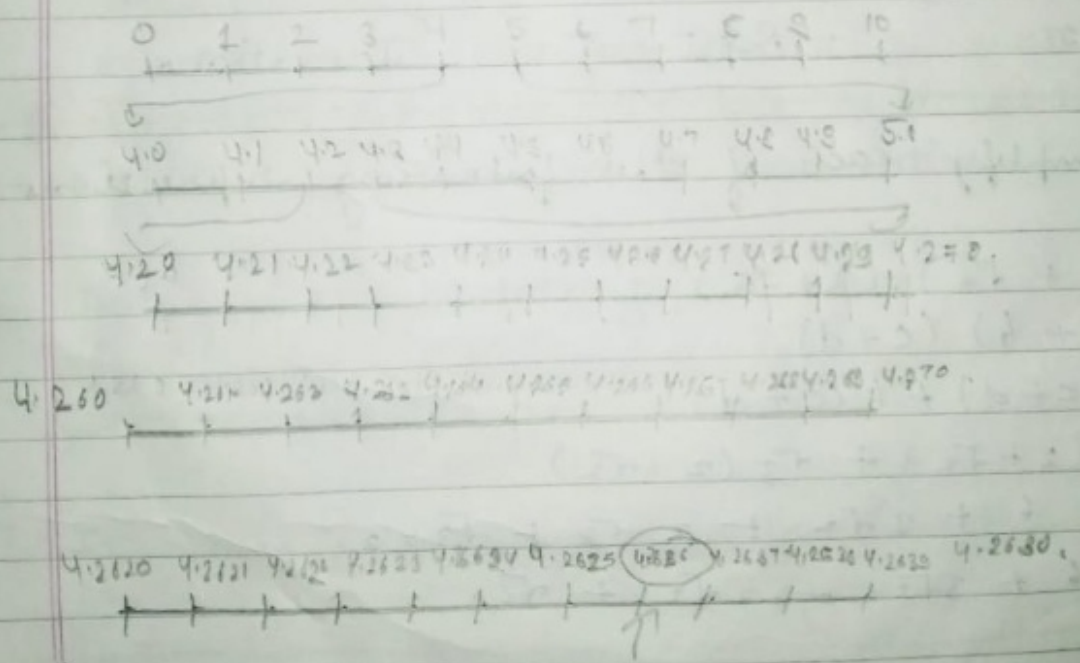
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Ex. - 1.4.

Q.1 Visualise 3.765 on the no. line using successive magnification.



Q.2 Visualize  $4.\overline{25}$  on the no line, upto 4 decimal places.



Ex - 15

Q.1: Classify the following no. as rational or irrational.

(i)  $2 - \sqrt{5}$  : R - I = I : Irrational

(ii)  $(3 + \sqrt{23} - \sqrt{23} - 3 + \sqrt{23} - \sqrt{23}) = \frac{3}{1} \frac{0}{9}$  : Rational

(iii)  $\frac{2\sqrt{7}}{2\sqrt{7}} = \frac{1}{1} \frac{1}{9}$  : Rational

(iv)  $\frac{1}{\sqrt{2}}$  :  $\frac{R}{I}$  : I : Irrational

(v)  $2\pi$  : R x I = I : Irrational

Q.2: Simplify each of the following expressions-

(i)  $(3 + \sqrt{3})(2 + \sqrt{2})$

Ans -  $(a + b)(c + d)$

$a(c + d) + b(c + d)$

$3(2 + \sqrt{2}) + \sqrt{3}(2 + \sqrt{2})$

$6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{3} \times 2$

$6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$

(ii)  $(3 + \sqrt{3})(3 - \sqrt{3})$

Ans =  $(a + b)(a - b) = a^2 - b^2$

$3^2 - (\sqrt{3})^2$

$9 - 3 = 6$



(iii)  $(\sqrt{5} + \sqrt{2})^2$   
 ans -  $(a+b)^2 = a^2 + 2ab + b^2$   
 $= (\sqrt{5})^2 + 2 \times \sqrt{5} \times \sqrt{2} + (\sqrt{2})^2$   
 $= 5 + 2 \times \sqrt{5} \times 2 + 2$   
 $= 5 + 2 + 2\sqrt{10}$   
 $= 7 + 2\sqrt{10}$

(iv)  $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$   
 ans -  $(a-b)(a+b) = a^2 - b^2$   
 $= (\sqrt{5})^2 - (\sqrt{2})^2$   
 $= 5 - 2 = 3$

Q. 3. Recall,  $\pi$  is defined as the ratio of all the circumference (say  $c$ ) of a circle to its diameter (say  $d$ ). That is  $\pi = \frac{c}{d}$ . This seems to contradict the fact that  $\pi$  is irrational. How will you solve this contradiction?

soln - 1:  $\pi = \frac{c}{d}$   $c$  and  $d$  are Rational

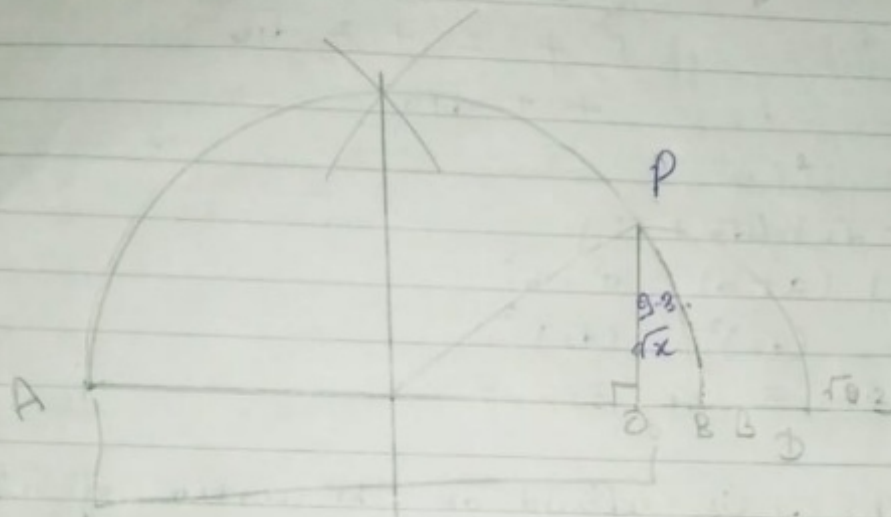
But if we calculate  $\frac{c}{d}$ , it gives  $\pi$

$\pi$  is irrational

It means either  $c$  or  $d$  or both are irrational

$$\frac{I}{R} = R \quad \frac{R}{I} = I \quad \frac{I}{I} = I$$

Q.4 Represent  $\sqrt{3}$  on the no. line.  
 Ans) construct  
 Proof



let  $x = 9.3$

Apply Pythagoras theorem  
 & cos

$$H^2 = c^2 + OP^2$$

$$CP^2 - OC^2 = OP^2$$

$$OP^2 = CP^2 - OC^2$$

$$CP = CB + \frac{3x+1}{2}$$

$$OC = \frac{x-1}{2}$$

$$OP^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2$$

$$OP^2 = \left[\frac{x^2 + 2x + 1}{4}\right] - \left[\frac{x^2 - 2x + 1}{4}\right]$$

Q.5

$$OP = \sqrt{\frac{x^2 + 2x + 1}{4} - \frac{x^2 - 2x + 1}{4}}$$

$$OP = \sqrt{\frac{4x}{4}}$$

$$OP = \sqrt{x}$$

Q.5 Rationalize the denominators of the following-

(i)  $\frac{1}{\sqrt{7}}$

$$\text{Ans. } \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \times 2} = \frac{\sqrt{2}}{2} \quad \text{Ans.}$$

$$(ii) \frac{1}{\sqrt{7} - \sqrt{6}}$$

$$\text{Ans. } \frac{1}{\sqrt{7} - \sqrt{6}} \times \frac{\sqrt{7} + \sqrt{6}}{\sqrt{7} + \sqrt{6}} = \frac{\sqrt{7} + \sqrt{6}}{(\sqrt{7})^2 - (\sqrt{6})^2}$$
$$(a+b) \cdot a^2 - b^2$$

$$= \frac{\sqrt{7} + \sqrt{6}}{7 - 6} = \frac{\sqrt{7} + \sqrt{6}}{1}$$

$$\sqrt{7} + \sqrt{6} \quad \text{Ans.}$$

$$(iii) \frac{1}{\sqrt{5} + \sqrt{2}}$$

$$\text{Ans. } \frac{1}{\sqrt{5} + \sqrt{2}} \times \frac{\sqrt{5} - \sqrt{2}}{\sqrt{5} - \sqrt{2}} = \frac{\sqrt{5} - \sqrt{2}}{(a+b)(a-b) = a^2 - b^2}$$

$$= \frac{\sqrt{5} - \sqrt{2}}{(\sqrt{5})^2 - (\sqrt{2})^2}$$

$$\frac{\sqrt{5} - \sqrt{2}}{5 - 2} = \frac{\sqrt{5} - \sqrt{2}}{3} \quad \text{Ans.}$$

$$(iv) \frac{1}{\sqrt{7} - 2}$$

Ans:  $\frac{1}{\sqrt{7}-2} \times \frac{\sqrt{7}+2}{\sqrt{7}+2}$

$$\frac{\sqrt{7}+2}{(\sqrt{7})^2 - 2^2}$$

$$= \frac{\sqrt{7}+2}{\sqrt{7}-4} = \frac{\sqrt{7}+1}{3} = \text{Ans.}$$

Ex. - 1.1.

Q.7 Find :

(i)  $64^{\frac{1}{2}}$

Ans:  $64^{\frac{1}{2}}$

$$= (2 \times 2 \times 2 \times 2 \times 2 \times 2)^{\frac{1}{2}}$$

$$= \left(\frac{2^6}{3}\right)^{\frac{1}{2}}$$

$$= 2^{6 \times \frac{1}{2}}$$

$$= 2^3 = 2 \times 2 \times 2 = 8 = \text{Ans.}$$

(ii)  $32^{\frac{1}{5}}$

Ans:  $32^{\frac{1}{5}}$

$$= (2^5)^{\frac{1}{5}}$$

$$= 2^{5 \times \frac{1}{5}}$$

$$= 2^1 = 2 = \text{Ans.}$$

(iii)  $125^{\frac{1}{3}}$

Ans:  $125^{\frac{1}{3}}$

$$\frac{1}{5} = \text{Ans.}$$

Q:3 Simplify

$$\begin{aligned} \text{(i)} \quad & 2^{\frac{2}{3}} \times 2^{\frac{1}{5}} \\ & = 2^{\frac{2}{3}} \times 2^{\frac{1}{5}} \\ & = 2^{\left(\frac{2}{3} + \frac{1}{5}\right)} \\ & = 2^{\left(\frac{2 \times 5 + 1 \times 3}{15}\right)} \\ & = 2^{\left(\frac{10 + 3}{15}\right)} \\ & = 2^{\left(\frac{13}{15}\right)} = \text{Ans.} \end{aligned}$$

$$\text{(ii)} \quad \left(\frac{1}{3^2}\right)^7$$

$$= \left(\frac{1}{3^2}\right)^7$$

$$\frac{1^7}{3^{2 \times 7}} = \frac{1}{3^{14}} = \text{Ans.}$$

$$\text{(i)} \quad \frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$$

$$= \frac{11^{\frac{1}{2}}}{11^{\frac{1}{4}}}$$

$$11^{\left(\frac{1}{2} - \frac{1}{4}\right)}$$

$$= 11^{\left(\frac{1 \times 2 - 1 \times 1}{4}\right)}$$

$$= 11^{\left(\frac{2 - 1}{4}\right)}$$

$$= 11^{\left(\frac{1}{4}\right)} = \text{Ans}$$